HILLSBOROUGH TOWNSHIP SCHOOL DISTRICT

MATHEMATICS CURRICULUM

AP Calculus BC

July, 2020

Course Overview
AP Calculus BC

AP Calculus BC is offered to students who have been successful in AP Calculus AB. during this course students explore, visualize, analyze, and calculate according to the topics referenced by the *AP Calculus Course Description*, and are also exposed to the practical applications of the principles learnt to various areas of science and engineering. Concepts are investigated analytically, graphically, and numerically. Students are expected to relate various representations to each other. The teacher of this course has also developed additional long-term assignments to aid students in preparation for taking the AP Exam. Students gain a thorough understanding of both Differential Calculus and Integral Calculus through various exercises provided by the course textbook as well as classroom investigations and activities as outlined below.

Throughout the course students will use a variety of strategies and tools to gain a solid understanding of the material. Technology which includes computer work, graphing calculators and exploring web based sites will enhance the educational experience by allowing lessons to become more interactive and inquiry-based.

The course is structured around the New Jersey Student Learning Standards. The curriculum will reflect various teaching strategies and offer opportunities for enrichment and reinforcement based on individual need.

Hillsborough Township Public Schools
AP Calculus BC Mathematics Curriculum

Unit Title: 1 Justification, Symbolic Analysis, & Proof Methods	**Timeframe/Pacing:** 5 days

Essential Questions

- How can we comprehensively communicate mathematics in writing?
- How can we prove or disprove a statement?
- How can we solve a problem generally and symbolically?

Enduring Understandings

- The contrapositive of a conditional statement holds equivalent truth value.
- A single counterexample disproves a statement, but a single example does not prove a statement.
- Letters or symbols can help represent the general case and mathematical operations can be applied to prove a general statement.

Standards Taught and Assessed

- F-TF.C.9 Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
- G-CO.C.10 Prove theorems about triangles.
- G-GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.
- A-APR.C.4 Prove polynomial identities and use them to describe numerical relationships.

Highlighted Interdisciplinary Connections

- ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
- ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
- Computer Science & Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions

Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).

Hillsborough Township Public Schools
AP Calculus BC Mathematics Curriculum

Pre-Assessment • Write the inverse, converse and contrapositive of a conditional statement. • Justify a relative extreme point using a graph of the derivative of a function.		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		
Student Learning Objectives: We are learning to/that...	**Student Strategies (Mathematical Practices)**	**Formative Assessment**	**Activities and Resources**	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
Simplify numeric and symbolic factorial expressions.	SMP 2 Reason abstractly and quantitatively.	Simplify: • $\frac{8!}{2!6!}$ • $\frac{(n+1)!}{(n-1)!}$	Activity: Rolling dice to count combinations and write equivalent calculation using combination formula, simplifying to arrive at valid response.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Write and evaluate the equivalence of different forms of conditional statements.	SMP 3 Construct viable arguments and critique the reasoning of others.	Write the inverse, converse and contrapositive of the given statement and identify which is/are true: If $x > 0$, then $x + 1 > 0$.	Conditional statement card sorting (Google Slides)	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use Proof by Contradiction to disprove a given statement.	SMP 3 Construct viable arguments and critique the reasoning of others.	Show a counterexample to disprove each statement: If $a < b$, then $a^2 < b^2$. If $a < b$, then $\frac{1}{a} < \frac{1}{b}$.	Students create statements that can be disproven using Proof by Contradiction.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Use Proof by Contraposition to prove a given statement.	SMP 3 Construct viable arguments and critique the reasoning of others.	Prove the given statement using Proof by Contraposition: If n^2 is even, then n is even.	Look at list of known Calculus Theorems from prior course → Write contrapositive of each proof and show an example.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use Proof by Exhaustion to prove a given statement.	SMP 5 Use appropriate tools strategically.	Prove the following statement using Proof by Exhaustion: Given $n \in \mathbb{N}$, if $2 \leq n \leq 5$, then 4 is not divisible by $n^2 + 2$.	Extension: The Method of Exhaustion and the Limit Process	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use Direct Proof to prove a given statement.	SMP 6 Attend to precision.	Let n be an integer. Show that: If n is even, then $3n$ is even. If n is odd, then $3n$ is odd.	Students demonstrate their work to others to evaluate clarity and thoroughness.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use Proof by Mathematical Induction to prove a given statement.	SMP 7 Look for and make use of structure.	Use Proof by Mathematical Induction to show that $1 \cdot 3 \cdot 5 \cdots (2k-1) = \frac{(2k)!}{2^k k!}$	Small group: Miscellaneous Inductive Proofs	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Write and justify mathematical ideas clearly and comprehensively.	SMP 3 Construct viable arguments and critique the reasoning of others.	Given a graph of f', determine value(s) of x where there exist relative extrema and points of inflection.	Article: On the Role of Sign Charts Writing Mathematics p93-102 (Proofs and Fundamentals, Bloch)	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Benchmark Assessment
- Not Applicable

Summative Assessment(s)
- Proofs and Fundamentals Quiz

Performance Assessment:
- Collaborative Challenge Proofs

Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
- Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Hillsborough Township Public Schools
AP Calculus BC Mathematics Curriculum

Unit Title: 2 Differentiation	**Timeframe/Pacing:** 14 days

Essential Questions

- How can we find the rate of change of a quantity?
- How does the rate of change of one quantity affect the rate of change of another?
- How can we visualize the rate of change of a quantity and what does that tell us?

Enduring Understandings

- Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals.
- Recognizing that a function's derivative may also be a function allows us to develop knowledge about the related behaviors of both.

Standards Taught and Assessed

- F-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Highlighted Interdisciplinary Connections

- ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
- ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
- Computer Science & Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions

Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).

Pre-Assessment • Evaluate indeterminate forms of limits analytically. • Match the limit definition of a derivative with the derivative of given functions. • Determine how the rate of change of one quantity affects the rate of change of another quantity (Chain Rule). • Make a graph that depicts the instantaneous rates of change of another graph at each point.		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		
Student Learning Objectives: We are learning to/that...	**Student Strategies (Mathematical Practices)**	**Formative Assessment**	**Activities and Resources**	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
Relate the limit definition of a derivative of a function and its derivative.	SMP 2 Reason abstractly and quantitatively.	Given $f'(a) = \lim_{h\to 0} \frac{(2+h)^2-2^2}{h}$, find $f(x)$, $f'(a)$.	Teacher-led derivation of definition of derivative with tiered examples of recognizing patterns between function derivative and definition.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Understand the relationship between differentiability and continuity.	SMP 2 Reason abstractly and quantitatively.	Given the graph of the derivative of a function, determine points at which the function must be continuous and/or differentiable.	Matching activity between graphs and their derivatives; Additional challenge to construct equation of derivative from graph with attention to discontinuities in derivative.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Apply differentiation rules (constant rule, power rule, constant multiple rule, sum and difference rule, sine and cosine, product rule, quotient rule, chain rule).	SMP 5 Use appropriate tools strategically.	<table><tr><td>x</td><td>f</td><td>f'</td><td>g</td><td>g'</td></tr><tr><td>2</td><td>5</td><td>4</td><td>-3</td><td>9</td></tr><tr><td>4</td><td>2</td><td>2</td><td>-2</td><td>3</td></tr></table> Find $\frac{d}{dx}[f \cdot g]_{x=3}$; $\frac{d}{dx}[f(f(2x)]_{x=3}$	Teacher-led derivation of derivative rules; Scaffolded examples and Think-Pair-Share of solutions.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use derivatives to find rates of change.	SMP 1 Make sense of problems and persevere in solving them.	A diver jumps from a platform that is 32 feet above the water with an initial velocity of 16 feet per second. When does the diver hit the water and with what velocity?	Group or paired challenge problems	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Evaluate higher-order derivatives.	SMP 7 Look for and make use of structure.	Suppose $h(x) = x \cdot f(x)$. Find $h'(x)$, $h''(x)$, $h'''(x)$, and then develop a general rule for the nth derivative of $h(x)$.	Guided examples to elicit pattern recognition	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use implicit differentiation to find the derivative of a function.	SMP 7 Look for and make use of structure.	Determine whether the two families of curves are orthogonal: $xy = C$ $x^2 - y^2 = K$	Teacher-led discussion of process and connections to chain rule and explicit forms; Desmos illustrations of orthogonal relationships	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or

				504 plan.
Find a related rate and use related rates to solve real-life problems.	SMP 4 Model with mathematics.	A particle moves around $x^2 + y^2 = 9$ in a counter-clockwise direction. At the time when it is at a position where $x = -2$ and $\frac{dx}{dt} = 2\frac{cm}{s}$, what is the rate of change of its distance from the fixed point on the same graph where $y = 3$?	Geometer's Sketchpad Calculus In Motion visualizations to demonstrate related rates; Group or paired problems requiring strategic thinking and modeling.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment • Not Applicable		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		
Summative Assessment(s) • Unit 2 Test **Performance Assessment**: • Related Rates Task (ie: Pizza Sauce Dispenser)				

Hillsborough Township Public Schools
AP Calculus BC Mathematics Curriculum

Unit Title: 3 Applications of Differentiation	**Timeframe/Pacing:** 17 days

Essential Questions

- What do derivatives tell us about the behavior of values in a graph?
- How can we piece together many different numerical characteristics to sketch a picture of a numerical relationship?
- How can we use derivatives in a context to make improvements of systems?

Enduring Understandings

- The first derivative indicates direction and the second derivative indicates shape
- The combination of skills acquired through Math Analysis and Differential Calculus describe the trends, shape, direction and key features of a graph.
- Optimization is a process making use of derivatives in order to help find maximal or minimal values to make systems better.

Standards Taught and Assessed

- F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases
- F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- G-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Highlighted Interdisciplinary Connections

- ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
- ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
- Computer Science & Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
- Computer Science & Design Thinking: 8.1.2.AP.4: Break down a task into a sequence of steps.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions

Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).

Pre-Assessment	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
• Draw a graph of a function given information about its first and second derivatives, end behavior expressed as limits. • Find the maximal value of a function on a closed interval. • Model an objective and constraint as a function of one variable and find its extrema using technology.	• Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Student Learning Objectives: We are learning to/that...	**Student Strategies (Mathematical Practices)**	**Formative Assessment**	**Activities and Resources**	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
Find absolute and relative extrema on an open or closed interval.	SMP 2 Reason abstractly and quantitatively.	Find all absolute and relative extrema of $y = \frac{sinu}{2+cosu}$ on $[0, \pi]$.	Teacher-led discussion of definitions with visual examples; tiered exploration to elicit general strategies	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Apply Rolle's Theorem and Mean Value Theorem	SMP 3 Construct viable arguments and critique the reasoning of others.	Determine reasoning to support the fact that the function $f(x) = x^{2/3}$ fails to satisfy the conclusion of the Mean Value Theorem on the interval [-8, 8].	Geometer's Sketchpad or Desmos visualization demos to illustrate theorems.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Determine intervals on which a function is increasing or decreasing.	SMP 2 Reason abstractly and quantitatively.	Determine where $f(x) = -\frac{1}{4}sin2x + cosx$ is increasing or decreasing.	Recall method from previous course and apply to more advanced functions.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Apply the First Derivative Test to find relative extrema of a function.	SMP 3 Construct viable arguments and critique the reasoning of others.	Given $y = f(x)$ and $y' = (x-1)^2(x-2)$, determine the x-values at which y has relative extrema and justify your answer.	Read: On the Role of Sign Charts (CollegeBoard) and illustrate understanding of valid justification methods by applying to new problems	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Determine intervals on which a function is concave upward or downward.	SMP 2 Reason abstractly and quantitatively.	Determine where $f(x) = -\frac{1}{4}sin2x + cosx$ is concave up or concave down.	Visually elicit definition of concavity and relate to second derivative data by describing "how the slope changes."	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Apply the Second Derivative Test to find relative extrema of a function.	SMP 3 Construct viable arguments and critique the reasoning of others.	Given $y = f(x)$ and $y' = (x-1)^2(x-2)$, determine the x-values at which y has relative extrema and justify your answer using the Second Derivative Test.	Continue using methods from: On the Role of Sign Charts (CollegeBoard).	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Determine end behavior of a function using limits at infinity.	SMP 7 Look for and make use of structure.	Determine the end behavior of $y = 4x - \sqrt{16x^2 - x}$.	Warm-up using previously studied examples and tiered examples to challenge incorporation of advanced analytic methods. Emphasize examples where arithmetic between values of infinity is counter-intuitive.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Analyze and sketch the graph of a function.	SMP 2 Reason abstractly and quantitatively.	Create a curve sketch for $f(x) = -\frac{1}{4}sin2x + cosx$ on $[0, \pi]$ using derivative and second derivative information.	Group activity to practice varied curve sketching problems.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Solve applied minimum and maximum problems.	SMP 1 Make sense of problems and persevere in solving them.	A line with slope m passes through (0, 4). Write the distance d between the line and point (3, 1) as a function of m. Determine the value of m for which the distance is minimized/maximized.	Group work on varied optimization problems. Class-discussion on using different methods or models based on group responses.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use Newton's Method to approximate solutions to an equation.	SMP 8 Look for and express regularity in repeated reasoning.	Use Newton's Method to approximate the zero(s) of $f(x) = x - 2\sqrt{x+1}$ until two successive approximations differ by less than 0.001.	Teacher-led discussion on Newton-Raphson Method. Desmos visualization to illustrate nature of derivation and accuracy.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

<table>
<tr><td>Find the differential of a given function.</td><td>SMP 8 Look for and express regularity in repeated reasoning.</td><td>Approximate the value of $\sqrt[3]{26}$ using a linear approximation.</td><td>Teacher-led derivation of method of linear approximation with tiered examples.</td><td>Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.</td></tr>
<tr><td colspan="2">Benchmark Assessment
• Not Applicable</td><td colspan="3" rowspan="2">Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
• Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.</td></tr>
<tr><td colspan="2">Summative Assessment(s)
• Unit 3 Test
Performance Assessment:
• Optimization Task including aspects of curve sketching and linear approximation. (ie: Halloween Ghost Detective Activity)</td></tr>
</table>

Hillsborough Township Public Schools
AP Calculus BC Mathematics Curriculum

Unit Title: 4 Integration	**Timeframe/Pacing:** 14 days

Essential Questions

- How can we measure the space under a curve?
- What can the area under a function tell us, in context?
- How can we reverse derivative rules?
- What is the relationship between Differential and Integral Calculus?

Enduring Understandings

- A series of rectangles can be used to approximate the area under a curve.
- The area under a rate function represents the amount of change of that quantity over time.
- An antiderivative can be found by reversing the algorithm for the power rule and/or chain rule given a certain structure.

Standards Taught and Assessed

- F-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- F-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Highlighted Interdisciplinary Connections

- ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
- ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
- Computer Science & Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions

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Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).

Pre-Assessment

- Approximate the area under $f(x) = x^2 + 1$ on [0, 2] using a series of rectangles.
- Determine the distance travelled by a particle whose velocity is $v(t) = t^3 - 3t + 1$ on [0, 4]

Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)

- Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Student Learning Objectives: We are learning to/that...	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
Write the general solution and particular solution of a differential equation.	SMP 7 Look for and make use of structure.	Solve $f''(x) = 2cosx$, $f'(0) = 4$, $f(0) = -5$	Teacher-led discussion to elicit both visual and analytical connections between a function's derivative and the function with a vertical translation.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find antiderivatives either by pattern recognition (basic antiderivatives & chain rule) or by change of variables (u-substitution)	SMP 7 Look for and make use of structure.	Integrate: $\int \frac{x^4+8}{x^2} dx$ $\int cos^3 x sinx dx$	Small group or whole class activity to identify general rules for antiderivatives of given functions.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use sequences and series to represent numerical patterns.	SMP 2 Reason abstractly and quantitatively.	Write an expression for the left endpoint, right endpoint, and midpoint of the ith subinterval: [-1,-.4]	Teacher-led discussion with examples of each method. Think-pair-share multiple representations of valid solutions.	Extended time, use of calculator, challenge work and specific other accommodations/modificat

		[-.4, .2] [.2, .8] [.8, 2.4] [2.4, 3]		ions per a student's IEP or 504 plan.
Find the area of an enclosed region using the limit definition.	SMP 8 Look for and express regularity in repeated reasoning.	Find the area under the graph $f(x) = x^3 + 1$ on [1, 2] using the limit process.	Scaffolded activity to develop limit process based on n rectangles and generalized sequences.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Apply the definition of a Riemann sum.	SMP 3 Construct viable arguments and critique the reasoning of others.	Write the following expression as a definite integral: $\lim_{n\to\infty} \sum_{i=1}^{n} 3\left(\frac{i}{n}+2\right)^2\left(\frac{1}{n}\right)$	Teacher-led discussion of definition; Varied examples and compare multiple representations of equivalent solutions	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Evaluate a definite integral using limits and geometric formulas.	SMP 1 Make sense of problems and persevere in solving them.	Evaluate: $\int_{-3}^{3} \sqrt{9-x^2}dx$	Recall basic geometric formulas and apply limit definition to combinations of shapes.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Evaluate a definite integral using the properties of definite integrals.	SMP 7 Look for and make use of structure.	Given $\int_0^3 f(x)\,dx = 2$, $\int_3^5 f(x)\,dx = -3$, find: $\int_0^5 2f(x)\,dx$	AP Multiple Choice questions with multiple representations of this concept.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Evaluate a definite integral using the Fundamental Theorem of Calculus.	SMP 7 Look for and make use of structure.	Evaluate: $\int_{-1}^{2} \left\|x^2 + 2x - 3\right\| dx$	Teacher-led discussion focus on visualization leading to analytical representation using known properties.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the average value of a function over a closed interval by using the Mean Value Theorem for Integrals.	SMP 2 Reason abstractly and quantitatively.	Given $v(t) = t^3 - 10t^2 + 29t - 20$, find the average velocity of the particle during the first 5 seconds.	Geometer's Sketchpad Calculus in Motion visualizations of MVT. Paired problem solving.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Apply the Second Fundamental Theorem of Calculus.	SMP 7 Look for and make use of structure.	Determine where $R(x) = \int_{-3x}^{3} \sqrt{1+t}\,dt$ is increasing and decreasing.	Teacher-led discussion to elicit proof-by-example of validity of Theorem; Elicit general procedure for all cases.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Apply the Net Change Theorem.	SMP 1 Make sense of problems and persevere in solving them.	Given $v(t) = t^3 - 10t^2 + 29t - 20$, find the displacement and distance traveled from t=1 to t=5.	AP Calculus FRQ's focusing on rate-in and rate-out in context; Group problem solving.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Approximate a definite integral using the Trapezoidal Rule.	SMP 8 Look for and express regularity in repeated reasoning.	Write a trapezoidal approximation for $f(x) = x^2 + 1$ on [2, 6] with n=6, using sigma notation.	Paired or group applications using topography	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Approximate a definite integral using Simpson's Rule	SMP 8 Look for and express regularity in repeated reasoning	Write a Simpson's Rule approximation for $f(x) = x^2 + 1$ on [2, 6] with n=6, using sigma notation.	Paired or group applications using topography	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Benchmark Assessment • Not Applicable	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Summative Assessment(s) • Unit 4 Test **Performance Assessment**: • CollegeBoard FRQ Rate-in-Rate-out Problem	

Hillsborough Township Public Schools
AP Calculus BC Mathematics Curriculum

Unit Title: 5 Transcendental Functions	**Timeframe/Pacing:** 18 days

Essential Questions

- How can we find rates of change of transcendental functions?
- How can we find the area under a transcendental function?
- How do properties of inverse functions allow us to build connections and find otherwise unknown quantities?

Enduring Understandings

- Properties of exponents, logarithms and inverse functions allow us to use implicit differentiation to establish new relationships and models.
- The relative rate of change of two quantities is consistent with that of their derivatives.
- Trigonometric connections can be used to solve integrals.

Standards Taught and Assessed

- F-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.
- F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph
- F-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

Highlighted Interdisciplinary Connections

- ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
- ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
- Computer Science & Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions

Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings,

death, car accidents, illness).

Pre-Assessment • Differentiate basic exponential, logarithmic, and inverse trigonometric functions. • Express the relationship between the slopes on two functions which are inverses of each other. • Solve an elementary half-life problem.	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Student Learning Objectives: We are learning to/that...	**Student Strategies (Mathematical Practices)**	**Formative Assessment**	**Activities and Resources**	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
Differentiate Natural Logarithmic functions, using basic properties where appropriate.	SMP 7 Look for and make use of structure.	Differentiate $y = x\ln\sqrt{x}$.	Teacher-led discussion emphasizing visual and analytical definition of Natural Log function and application of Second Fundamental Theorem of Calculus	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Integrate functions resulting in a Natural Logarithmic form.	SMP 7 Look for and make use of structure.	Integrate $\int \frac{4}{1+\sqrt{5x}}dx$.	Student-led lesson or group activity with mixed integrals illustrating varied methods of solving and identifying patterns.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the derivative of an inverse function (without knowing the inverse function).	SMP 2 Reason abstractly and quantitatively.	Given $f(x) = \int_2^x \sqrt{1+t^2}dt$, find $(f^{-1})'(0)$.	Exploration of derivatives of two inverse functions to elicit rule.	Extended time, use of calculator, challenge work and specific other accommodations/modificat

				ions per a student's IEP or 504 plan.
Differentiate Exponential functions, making use of properties of exponents where appropriate.	SMP 7 Look for and make use of structure.	Differentiate: $g(t) = (e^{-t} + tan(e^t))^3$	Guided proof:: use of implicit differentiation to derive rule.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Integrate Exponential functions.	SMP 7 Look for and make use of structure.	Integrate: $\int \frac{e^{-3x}+2e^{2x}+3}{e^x} dx$	Student-led lesson or group activity with mixed integrals illustrating varied methods of solving and identifying patterns.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Differentiate and Integrate Exponential functions that have bases other than e.	SMP 7 Look for and make use of structure.	Differentiate: $y = log_5 \frac{4}{x^2\sqrt{1-x}}$ Solve: $y' = 3^{cosx} sinx$; $y(0) = \frac{\pi}{2}$	Students derive general rules using logarithmic differentiation and/or implicit differentiation.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Solve applications of Exponential and Logarithmic functions and their differential equations.	SMP 1 Make sense of problems and persevere in solving them.	A lake is stocked with 500 fish and its population at time t months is $p(t) = \frac{10{,}000}{1+19e^{-t/6}}$. Find the rate at which the fish population is increasing at $t = 5$ months, and when it is increasing most rapidly.	Use of graphing calculator or desmos to visualize trends in graph and justify analytic trends.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Apply L'Hopital's Rule to evaluate a limit.	SMP 3 Construct viable arguments and critique the reasoning of others.	Evaluate: $\lim_{x\to\infty} x^{1/x}$	Teacher-led discussion with tiered example of various indeterminate forms.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Differentiate Inverse Trigonometric Functions.	SMP 7 Look for and make use of structure.	Differentiate: $y = xarctan2x - \frac{1}{4}ln(1+4x^2$	Guided-proof activity to derive general rules.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Integrate Inverse Trigonometric Functions.	SMP 7 Look for and make use of structure.	Integrate: $\int \frac{x-2}{(x+1)^2+4}dx$	Student practice examples using various forms in which problems may appear.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Benchmark Assessment • AP Classroom - Unit 2-5 Multiple Choice Assessments	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Summative Assessment(s) • Unit 5 Test **Performance Assessment**: • Logistic Population Problem	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Hillsborough Township Public Schools
AP Calculus BC Mathematics Curriculum

Unit Title: 6 Differential Equations	**Timeframe/Pacing:** 11 days

Essential Questions

- What information can we gain from a differential equation?
- How can we model real-world phenomena using differential equations?
- How can we best approximate solutions to problems when it is not possible to solve analytically?

Enduring Understandings

- Solving a differential equation provides information about the quantity whose rate of change is described.
- Different methods can be used to solve differential equations depending on the classification.
- Tangent line approximation methods can be expanded to visualize and approximate solutions to differential equations with great accuracy.

Standards Taught and Assessed

- F-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.
- F-BF.A.b Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
- F-BF.A.c Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
- F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Highlighted Interdisciplinary Connections

- ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
- ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
- Computer Science & Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
- Computer Science & Design Thinking: 8.1.2.AP.4: Break down a task into a sequence of steps.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions

Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).

Pre-Assessment	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
• Solve basic differential equations • Draw a basic slope field graph • Use Newton's Method to approximate a zero	• Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Student Learning Objectives: We are learning to/that...	**Student Strategies (Mathematical Practices)**	**Formative Assessment**	**Activities and Resources**	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
Verify solutions to a differential equation and find particular solutions.	SMP 6 Attend to precision.	Verify that the function $y = -\cos x \cdot \ln\lvert\sec x + \tan x\rvert$ is a solution to $y'' + y = \tan x$	Use of Desmos to visualize connections after validating analytically.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Create and match slope fields for a given differential equation.	SMP 5 Use appropriate tools strategically.	Draw a slope field for $\frac{dy}{dx} = xy - x^2$.	Provide pre-dotted slope field blanks; Use of a variety of examples of curves from implicit differentiation to elicit trends.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use Euler's Method to approximate solutions of differential equations.	SMP 7 Look for and make use of structure.	Compare the approximation given by Euler's method for $\frac{dy}{dx} = y + \cos x$, from (0, 0) to $x_n = 1$ with $\Delta x = .1$ to the actual solution.	Video Clip: Hidden Figures CollegeBoard Differential Equations Resources	Extended time, use of calculator, challenge work and specific other accommodations/modificat

				ions per a student's IEP or 504 plan.
Use separation of variables to solve a differential equation.	SMP 4 Model with mathematics.	Solve $y' = \frac{lnx}{xy+xy^3}$.	Teacher-led review of known methods with tiered examples using more complex problems; visualization of solutions using Desmos.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Solve exponential growth and decay problems modeled by differential equations (including Newton's Law of Cooling).	SMP 1 Make sense of problems and persevere in solving them.	An object is removed from a 1500 degree furnace and placed in a room with a temperature of 80 degrees. One hour later the temperature is 1120 degrees. What is the temperature of the object after 6 hours?	Paired or group problem solving activity with analytical and graphical components.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Solve a Logistic Differential Equation (using basic partial fraction decomposition).	SMP 5 Use appropriate tools strategically.	Solve $\frac{dP}{dt} = 3P(1 - \frac{P}{100})$.	Teacher-led derivation showing method of partial fractions	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Identify key components of a situation from a Logistic Differential Equation.	SMP 7 Look for and make use of structure.	Given $\frac{dP}{dt} = 3P(1 - \frac{P}{100})$, Find the carrying capacity and the time at which the population is growing most rapidly.	Think-Pair-Share to define key components without solving.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Identify and solve homogeneous differential equations.	SMP 7 Look for and make use of structure.	Solve $(2x + 3y)dx = xdy$	Teacher-led discussion of definition and method of solving. Emphasis on multiple representations of solutions and visualization using Desmos to check solutions.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Identify and solve a First-Order Linear Differential Equation (FOLDE)	SMP7 Look for and make use of structure.	Solve $y' + 3y = e^{3x}$	Teacher-led discussion of classification method and method of solving. Verify solutions graphically and analytically to bridge unit topics.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Solve applications of FOLDE's.	SMP 1 Make sense of problems and persevere in solving them.	A tank contains 50 gallons of a solution composed of 90% water and 10% alc. A second 50%/50% solution is added at 4 gal/min while the well stirred tank is drained at 5 gal/min. How much alc. is in the tank after 10 minutes?	Paired practice problems to develop modeling and solving.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment • AP Classroom Unit 7 Assessment(s)		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		
Summative Assessment(s) • Unit 6 Test **Performance Assessment**: • Mixture & Logistic Population Problems		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		

Hillsborough Township Public Schools
AP Calculus BC Mathematics Curriculum

Unit Title: 7 Applications of Integration	**Timeframe/Pacing:** 9 days

Essential Questions

- How can you use integrals to measure quantities and accumulations?
- How can you use integrals to measure geometric objects?

Enduring Understandings

- A definite integral, by definition, is the sum of its integrand over a closed interval.
- Integrating length and area over a closed interval gives area and volume, respectively.

Standards Taught and Assessed

- G-GMD.A. Explain volume formulas and use them to solve problems

  1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

  2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

  3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

  B. Visualize relationships between two-dimensional and three-dimensional objects

  4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
- F-IF.C. Analyze functions using different representations

  7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- N-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Highlighted Interdisciplinary Connections

- ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
- ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
- Computer Science & Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions

Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).

Pre-Assessment

- Find the area between two curves.
- Find the volume of a solid of revolution about the x-axis

Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)

- Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Student Learning Objectives: We are learning to/that...	Student Strategies (Mathematical Practices)	Formative Assessment	Activities and Resources	Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
Find the area between two curves using integration.	SMP 6 Attend to precision. SMP 2 Reason abstractly and quantitatively.	Describe two different regions whose area is represented by $\int_{-2}^{2} [x^2 + 1 + x]dx$	Recall definition of a definite integral and Reimann Sum. Elicit general formula for area between two curves.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the volume of a solid of revolution using the disk method.	SMP 4 Model with mathematics	Use the disc method to prove the geometric formulas for the volume of a sphere and cone.	Use Geometer's Sketchpad Calculus in Motion Demos to visualize.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Find the volume of a solid of revolution using the washer method.	SMP 4 Model with mathematics.	Describe the solid whose volume is given by $\pi\int_0^a [b^2 - [b - f(x)]^2]dx$	Use Geometer's Sketchpad Calculus in Motion Demos to visualize.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the volume of a solid with known cross sections.	SMP 4 Model with mathematics.	The base of a solid of a solid is bounded by $y=x^3$, $y=0$ and $x=1$. Find the volume of the solid for each of the following cross sections (taken perpendicular to the y-axis): a)diamonds b)semicircles c) equilateral triangles	Use foam board to demonstrate and/or construct models of the solids.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the volume of a solid of revolution using the shell method.	SMP 4 Model with mathematics.	Find the volume of the solid generated by rotating the region bounded by $y = 6 - x^2$ and $y = x^2$ about the y-axis.	Use Geometer's Sketchpad Calculus in Motion Demos to visualize.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Determine when to use the shell method or washer method to find the volume of a solid of revolution..	SMP 3 Construct viable arguments and critique the reasoning of others.	Find the volume of the solid generated by rotating the region bounded by $y = \sqrt{x}$ and $y = x^2$ about the x-axis using both the Shell Method and Disc/Washer Method and	Paired practice problems with mixed methods; focus on multiple representations and verifying solutions though equality of integrals.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

		explain which method is preferable.		
Find the arc length of a smooth curve both with and without the use of technology.	SMP 7 Look for and make use of structure.	Find the arc length of $y = \frac{x^7}{14} + \frac{1}{10x^5}$ on $[1, 2]$	Teacher-led proof of arc length using integration using distance formula and definition of definite integral.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the area of a surface of revolution	SMP 4 Model with mathematics.	Find the lateral surface area of a frustum with diagonal length L, and radii r and R.	Wrap a coffee cup in paper and estimate the surface area using known geometric methods.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment • Not Applicable		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		
Summative Assessment(s) • Unit 7 Test **Performance Assessment**: • Spherical Drill Modeling				

Hillsborough Township Public Schools
AP Calculus BC Mathematics Curriculum

Unit Title: 8 Integration Techniques	**Timeframe/Pacing:** 17 days

Essential Questions

- How can we identify patterns to find antiderivatives?
- How can we use trigonometric properties and identities to change the structure of a problem?
- How can we quantify the accumulation of space over an infinite interval?

Enduring Understandings

- Reversing and reorganizing known derivative rules gives processes for finding antiderivatives.
- Using limits, together with integrals, we can measure infinite spaces.

Standards Taught and Assessed

- F-BF.A.1. Write a function that describes a relationship between two quantities.
- A-SSE.A.1b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Highlighted Interdisciplinary Connections

- ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
- ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
- Computer Science & Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions

Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).

Hillsborough Township Public Schools

AP Calculus BC Mathematics Curriculum

Pre-Assessment • Mixed Integrals using "basic integration rules"		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		
Student Learning Objectives: We are learning to/that...	**Student Strategies (Mathematical Practices)**	**Formative Assessment**	**Activities and Resources**	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
Find an antiderivative using integration by parts.	SMP 7 Look for and make use of structure.	$\int x^2 lnxdx$	Teacher-led derivation of Integration By Parts formula using known product rule.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use a tabular method to perform integration by parts.	SMP 5 Use appropriate tools strategically.	$\int x^3 sinxdx$	Think-pair-share to theorize patterns in repeated use of integration by parts.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Solve Trigonometric Integrals	SMP 7 Look for and make use of structure.	$\int \frac{cos^3x}{\sqrt{sinx}} dx$	Teacher-led discussion illustrating all possible cases; student participation to develop a complete list of cases	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Use trigonometric substitution to find integrals	SMP 7 Look for and make use of structure.	$\int \frac{dx}{\sqrt{4x^2+1}}$	Teacher-led discussion to illustrate method followed by tiered examples.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use partial fraction decomposition with linear and quadratic factors to integrate rational functions	SMP 7 Look for and make use of structure.	$\int \frac{5x^2+20x+6}{x(x+1)^2} dx$	Teacher-led discussion to illustrate possible cases. Focus on connection to adding fractions with different combinations of common factors in denominator to make comparisons.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Evaluate an improper integral that has either an infinite limit of integration or an infinite discontinuity.	SMP 2 Reason abstractly and quantitatively.	$\int_0^\infty x^2 e^{-x} dx$	Review basic limits and L'Hopital's Rule for indeterminate forms; Illustrate connection to a convergent infinite series for distinct values.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use a comparison test to determine the convergence of an improper integral.	SMP 2 Reason abstractly and quantitatively.	$\int_0^\infty \frac{2}{1+x^3} dx$	Visual exploration and connection to Squeeze Theorem of convergent and divergent integrals	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment • AP Classroom Unit 8 Assessment		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		

Summative Assessment(s) • Unit 8 Test **Performance Assessment**: • Cumulative Task involving integrals from this unit, Euler's Method, Slope Fields	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Hillsborough Township Public Schools
AP Calculus BC Mathematics Curriculum

Unit Title: 9 Series	**Timeframe/Pacing:** 25 days

Essential Questions

- How can we represent numerical patterns?
- How can the sum of infinitely many discrete terms be a finite value or represent a continuous function?
- How can we solve questions of transcendental functions using series?

Enduring Understandings

- Applying limits may allow us to determine the finite sum of infinitely many terms.
- An infinite series may represent a function and therefore allow us to solve otherwise impossible questions.

Standards Taught and Assessed

- F-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.
- F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- F-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- F-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Highlighted Interdisciplinary Connections

- ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
- ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
- Computer Science & Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions

Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).

Pre-Assessment	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
• Find the *n*th term of an arithmetic series. • Find the sum of an infinite geometric series. • Determine if an infinite series converges based on its improper integral equivalent. • Write an infinite series as a Riemann Sum.	• Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Student Learning Objectives: We are learning to/that...	**Student Strategies (Mathematical Practices)**	**Formative Assessment**	**Activities and Resources**	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
Determine whether a sequence converges or diverges.	SMP 2 Reason abstractly and quantitatively.	Determine whether the sequence with the given *n*th term converges or diverges: $a_n = \frac{1+(-1)^n}{n^2}$	Connection to known limit methods; comparison of discrete vs. continuous-valued functions	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Write a formula for the *n*th term of a sequence.	SMP 7 Look for and make use of structure.	Write an explicit expression for the *n*th term of the sequence: $2,\ 1+\frac{1}{2},\ 1+\frac{1}{3},\ 1+\frac{1}{4},\ldots$	Think-Pair-Share tiered examples of sequence building	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use properties of monotonic and bounded sequences.	SMP 2 Reason abstractly and quantitatively.	Determine whether the sequence with the given *n*th term is monotonic and/or bounded showing	Teacher-led discussion; elicit criteria through visualization of examples	Extended time, use of calculator, challenge work and specific other accommodations/modificat

		proper justification: $a_n = \frac{1+(-1)^n}{n^2}$		ions per a student's IEP or 504 plan.
Use properties of infinite geometric sequences.	SMP 2 Reason abstractly and quantitatively.	Find the sum of the series: $\sum_{n=0}^{\infty}(-\frac{1}{5})^n$	Teacher-led derivation of partial sum and infinite sum of a geometric series formulae Article: Cantor's Disappearing Table	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Write a series in telescoping form and determine its sum.	SMP 7 Look for and make use of structure.	Find the sum of the series: $\sum_{n=1}^{\infty}\frac{4}{n(n+2)}$	Discovery activity: Students graph explicit values of series and then, given a hint, expand partial fraction decomposition and graph terms of series	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use the *n*th-term test for divergence	SMP 6 Attend to precision.	Determine the convergence or divergence of the series: $\sum_{n=1}^{\infty}\frac{n+1}{2n-1}$	Connect definitions of bounded and monotonic to create conditions for *n*th-term test.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use the integral test to determine whether a series converges or diverges.	SMP 7 Look for and make use of structure.	Confirm whether the Integral Test is applicable, and if so, use it to determine convergence of the series: $\sum_{n=1}^{\infty}\frac{n}{n^4+1}$	Teacher-led graphical exploration: write a Riemann sum for a function that is identical to a given series. Elicit conditions and conclusions for integral test.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Use properties of *p*-series and the harmonic series to determine convergence.	SMP 7 Look for and make use of structure.	Determine the convergence of the *p*-series: $1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\ldots$	Think-pair-share to elicit generalization of integral test into *p*-series test	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use the Direct Comparison Test to determine whether a series converges or diverges.	SMP 3 Construct viable arguments and critique the reasoning of others.	Confirm whether the Direct Comparison Test is applicable, and if so, use it to determine convergence of the series: $\sum_{n=1}^{\infty}\frac{1}{2+\sqrt{n}}$	Connection to comparison test for improper integrals	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use the Limit Comparison Test to determine whether a series converges or diverges.	SMP 3 Construct viable arguments and critique the reasoning of others.	Confirm whether the Limit Comparison Test is applicable, and if so, use it to determine convergence of the series: $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2+1}$	Student exploration to determine which types of series the test applies to and which give inconclusive results.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use the Alternating Series Test to determine whether a series converges or diverges.	SMP 3 Construct viable arguments and critique the reasoning of others.	Determine the convergence or divergence of the series: $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n}{3n+2}$	Teacher-led discussion with guided notes with visual depictions of alternating series to elicit conditions and conclusions of test	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use the Alternating Series Remainder to approximate the sum of an alternating series.	SMP 6 Attend to precision.	Determine the number of terms required to approximate the sum of the	Visual exploration of Alternating Series Error conclusion using graphed partial series.	Extended time, use of calculator, challenge work and specific other accommodations/modificat

		series with an error of less than 0.001 $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n}{3n+2}$		ions per a student's IEP or 504 plan.
Classify a convergent series as absolutely or conditionally convergent.	SMP 3 Construct viable arguments and critique the reasoning of others.	Determine whether the series converges absolutely, conditionally, or diverges: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n}{3n+2}$	Teacher-led discussion illustrating different cases. Extension reading: Riemann's Rearrangement Theorem	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use the Ratio Test to determine whether a series converges or diverges.	SMP 3 Construct viable arguments and critique the reasoning of others.	Use the Ratio Test to determine convergence, if it applies: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}2^n}{n!}$	CollegeBoard Infinite Series Investigation (pg. 35-37)	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use the Root Test to determine whether a series converges or diverges.	SMP 3 Construct viable arguments and critique the reasoning of others.	Use the Root Test to determine convergence, if it applies: $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n$	Teacher-led discussion illustrating criteria for use of test; visual comparisons to known series	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Choose an appropriate convergence test to analyze a given series.	SMP 7 Look for and make use of structure. SMP 6 Attend to precision.	Determine the convergence or divergence of the series using any appropriate test: a) $\sum_{n=1}^{\infty} \frac{10}{3\sqrt{n^3}}$ b) $\sum_{n=1}^{\infty} \frac{2^n}{2n^2-1}$	Group practice with mixed series	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Find Taylor and Maclaurin polynomials for elementary functions.	SMP 4 Model with mathematics.	Find the 4th degree Taylor polynomial for $f(x) = lnx$ centered at $x = 4$, find $p_4(4.1)$ and compare its value to the known value of $ln(4.1)$.	Exploration activity in groups to derive standard form of a Taylor polynomial.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find and use the remainder of a Taylor polynomial. (Lagrange remainder & Taylor's Theorem)	SMP 6 Attend to precision.	Use Taylor's Theorem to obtain an upper bound for the error of the approximation of $ln(2.1)$ using a 4th degree Taylor polynomial.	Exploration Activity Guided examples	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Understand the definition of a power series.	SMP 2 Reason abstractly and quantitatively.	State where the power series is centered: $\sum_{n=0}^{\infty} \frac{(-1)^n(2n-1)}{2^n n!} x^n$	Teacher-led discussion: Define a power series and elicit student responses about characteristics	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the radius of a power series.	SMP 7 Look for and make use of structure.	Find the radius of convergence for the series: $\sum_{n=0}^{\infty} \frac{(-1)^n(2n-1)}{2^n n!} x^n$	Teacher-led discussion: connection from ratio test to convergence of power series	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Determine endpoint convergence and interval of convergence of a power series.	SMP 6 Attend to precision.	Find the interval of convergence of the series: $\sum_{n=0}^{\infty} \frac{(-1)^n(2n-1)}{2^n n!} x^n$	Teacher-led discussion: connection from ratio test to interval of convergence of power series	Extended time, use of calculator, challenge work and specific other accommodations/modificat

				ions per a student's IEP or 504 plan.
Differentiate and integrate a power series.	SMP 7 Look for and make use of structure.	Show that the function represented by the power series is a solution of the differential equation: $y = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ $y'' + y = 0$	Paired activity to work through differential equations problems; attention to switching bounds of series where necessary	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find a geometric power series that represents a function.	SMP 8 Look for and express regularity in repeated reasoning.	Find a power series for the function, centered at x = -3: $f(x) = \frac{5}{2x-3}$	CollegeBoard Infinite Series Special Focus Activity (pg. 11-16)	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Construct a power series using series operations.	SMP 8 Look for and express regularity in repeated reasoning.	Find a power series for: $f(x) = \frac{3x-1}{x^2-1}$	Guided exploration comparing intervals of convergence for related power series	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find a Taylor or Maclaurin series for a function.	SMP 8 Look for and express regularity in repeated reasoning.	Find the Maclaurin series for the function $f(x) = 2sin(x^3)$	CollegeBoard Infinite Series Investigation (pg. 32-35)	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Use Taylor series to solve integrated problems.	SMP 1 Make sense of problems and persevere in solving them.	Evaluate $\int_0^{1/2} \frac{\arctan x}{x}\,dx$ using a 6th degree Taylor polynomial.	Comparison activity for numerical approximation methods of integration and Taylor series	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment • AP Classroom Unit 10 Assessments (MC & FRQ)		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		
Summative Assessment(s) • Series Convergence Tests Assessment • Sequences and Series Assessment **Performance Assessment**: • Comparison of Integration Methods inclusive of Power Series and Taylor Series Representations		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		

Hillsborough Township Public Schools
AP Calculus BC Mathematics Curriculum

Unit Title: 10 Parametric & Polar Calculus	**Timeframe/Pacing:** 12 days

Essential Questions

- How can we model mathematical relationships with different systems?
- How can we use known Calculus concepts to solve problems in different coordinate systems?
- How can we convert from one mathematical coordinate system to another?

Enduring Understandings

- Derivatives allow us to solve real-world problems involving rates of change.
- Solving an initial value problem allows us to determine an expression for the position of an object in the plane.
- Definite integrals allow us to solve problems involving the accumulation of change in distance, area or volume over an interval.

Standards Taught and Assessed

- F-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship
- F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-BF.A.1 Write a function that describes a relationship between two quantities.

Highlighted Interdisciplinary Connections

- ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
- ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
- Computer Science & Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions

Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).

Pre-Assessment	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
• Plot points given in polar coordinates. • Determine the position of a particle at time t given parametric equations. • Find the arc length of a circle using integration.	• Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Student Learning Objectives: We are learning to/that...	**Student Strategies (Mathematical Practices)**	**Formative Assessment**	**Activities and Resources**	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
Sketch the graph of a curve given by a set of parametric equations.	SMP 1 Make sense of problems and persevere in solving them.	Sketch the curve, indicating orientation: $x = t^2 + t,\ y = t^2$	CollegeBoard Focus on Vectors (pg. 6-13)	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Eliminate the parameter in a set of parametric equations.	SMP 7 Look for and make use of structure.	Eliminate the parameter to obtain the standard rectangular form: $x = t^2 + t,\ y = t + 1$	Validate solutions using Desmos or graphing calculator to graph both forms.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find a set of parametric equations to represent a curve.	SMP 2 Reason abstractly and quantitatively.	Find two sets of parametric equations for the rectangular equation: $y = 6(x - 2)$	Validate solutions using Desmos or graphing calculator to graph both forms.	Extended time, use of calculator, challenge work and specific other accommodations/modificat

				ions per a student's IEP or 504 plan.
Find the slope of a tangent line and concavity to a curve given by a set of parametric equations.	SMP 3 Construct viable arguments and critique the reasoning of others.	Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $t = 3$ for $x = cos\theta$, $y = 4sin\theta$.	CollegeBoard Focus on Vectors (pg. 22-26)	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the speed of an object given by a set of parametric equations.	SMP 6 Attend to precision.	Determine the speed of an object whose parametric equations are $x = t^2 + t$, $y = t + 1$ at $t =$ 3.	CollegeBoard Focus on Vectors (pg. 27-29)	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the arc length of a curve given by a set of parametric equations.	SMP 7 Look for and make use of structure.	Find the arc length of the curve $x = t^2 + t$, $y = t + 1$ on [0, 2].	Teacher-led derivation of arc length formula from known formula in rectangular coordinates.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Plot points using the polar coordinate system.	SMP 3 Construct viable arguments and critique the reasoning of others.	Plot: $(8, \frac{\pi}{2})$ and write 3 other points that are located at the same place on the polar graphing plane.	Desmos Student Activity	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Convert between rectangular and polar	SMP 7 Look for and make use of structure.	Convert the polar equation to rectangular form: $r = -6csc\theta$	Validate solutions using Desmos or graphing calculator to graph both	Extended time, use of calculator, challenge work and specific other

coordinate forms. (equations and points)			forms.	accommodations/modifications per a student's IEP or 504 plan.
Sketch the graph of an equation given in polar form.	SMP 8 Look for and express regularity in repeated reasoning.	Sketch: a) $r = -6\csc\theta$ b) $r = 1 + \cos\theta$	Desmos Student Activity	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the slope of a tangent line to a polar graph.	SMP 6 Attend to precision.	Evaluate: $\frac{d}{dx}[1 + \cos\theta]_{\theta=\frac{\pi}{2}}$	Teacher-led discussion of conversion from polar equation to find $\frac{dy}{dx}$	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the area of a region bounded by one or more polar graphs.	SMP 1 Make sense of problems and persevere in solving them.	Find the area of the common interior of $r = -6\cos\theta$ and $r = 2 - 2\cos\theta$.	Teacher-led discussion bridging prior knowledge to applying integration to a new coordinate system	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Find the arc length of a polar graph.	SMP 7 Look for and make use of structure.	Find the arc length of the cardiod $r = 2 - 2\cos\theta$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$.	Teacher-led derivation of formula for arc length using parametric form	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Understand and express information as a vector	SMP 4 Model with mathematics.	Express the velocity of $x = t^2 + t,\ y = t + 1$ at $t = 3$ as a vector.	CollegeBoard Focus on Vectors (pg. 6-13)	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Determine a particular solution given a rate vector and initial conditions.	SMP 1 Make sense of problems and persevere in solving them.	Given $x' = 2t + 1$ and $y' = \sqrt{t}$, How far does the particle move, and where is it located at $t = 2$ if the initial position is (2, 4)?	CollegeBoard Focus on Vectors (pg. 39-42)	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment • AP Classroom Unit 9 Assessment		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		
Summative Assessment(s) • Unit 10 Test **Performance Assessment**: • AP Vector Free-Response Question		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		

Hillsborough Township Public Schools
AP Calculus BC Mathematics Curriculum

Unit Title: 11 Connections between Calculus Topics	**Timeframe/Pacing:** 15 days

Essential Questions

- How can limits be used to explore relationships between quantities?
- How can we find rates of change and what can they be used to measure?
- How can we find antiderivatives and what can they be used to measure?
- How can Calculus be used to model and solve real-world problems?

Enduring Understandings

- Limits can help us determine otherwise unknown quantities when real numbers fail to exist.
- Derivatives (and their rules and properties) find rates of change and help optimize systems.
- Integrals (and methods of integration) can be used to model systems and determine quantities of accumulation.
- Taylor series can be used to represent elementary functions and solve applied problems.

Standards Taught and Assessed

- F-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship
- F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-BF.A.1 Write a function that describes a relationship between two quantities.
- F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- G-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
- F-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.
- F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- G-GMD.A.3 Explain volume formulas and use them to solve problems. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- G-GMD.B.4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Highlighted Interdisciplinary Connections

- ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
- ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
- Computer Science & Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions

Social Emotional Learning Competencies

- 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).

Pre-Assessment	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
• Past AP Free-Response Collection	• Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Student Learning Objectives: We are learning to/that...	**Student Strategies (Mathematical Practices)**	**Formative Assessment**	**Activities and Resources**	**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)**
Use derivatives to describe rates of change of one variable with respect to another or using definite integrals to describe the net change in one variable over an interval.	SMP 1 Make sense of problems and persevere in solving them.	2018 Exam - Question #1 & #3	Emphasis on relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus. Past AP Calculus	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

			Free-Response Questions	
Use discrete values and limit concepts, definitions, formulas, and theorems in calculus, solve problems of continuity, differentiation, integration.	SMP 5 Use appropriate tools strategically.	2018 Exam - Question #4	Paired or group collaboration; switch and score using official rubrics; teacher-led discussion of best practices Past AP Calculus Free-Response Questions	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Analyze the behaviors of functions by relating limits to differentiation, integration, and relating each of these concepts to the others.	SMP 2 Reason abstractly and quantitatively.	2016 Exam - Question #4	Past AP Calculus Free-Response Questions	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Solve integrated problems involving particle motion, parametric, or polar coordinates.	SMP 1 Make sense of problems and persevere in solving them.	2018 Exam - Question #5	Past AP Calculus Free-Response Questions	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.

Solve integrated problems using Taylor Series, series convergence concepts, limits, derivatives and integrals	SMP 6 Attend to precision	2018 Exam - Question #6	Past AP Calculus Free-Response Questions	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment • Past AP Calculus BC Multiple Choice Assessment		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		
Summative Assessment(s) • See Benchmark **Performance Assessment**: • Graded AP Free Response Tasks		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		

Hillsborough Township Public Schools
AP Calculus BC Mathematics Curriculum

Unit Title: 12 Additional Topics in Calculus	**Timeframe/Pacing:** 23 days

Essential Questions

- How can Calculus be used to solve advanced physical problems?
- What bridges exist between Calculus and other scientific subjects?

Enduring Understandings

- Problems can be solved multiple ways, using multiple representations in many cases.
- Numerical Approximation methods are often extremely accurate and computers can facilitate the processing of numbers.

Standards Taught and Assessed

- F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship
- F-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.
- F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Highlighted Interdisciplinary Connections

- ELA: SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
- ELA: SL.11-12.5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
- Computer Science & Design Thinking: 8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
- Computer Science & Design Thinking: 8.1.2.AP.3: Create programs with sequences and simple loops to accomplish tasks.
- Computer Science & Design Thinking: 8.1.2.AP.4: Break down a task into a sequence of steps.
- Science: HS-PS2-2 Use mathematical representations to support the claim that the total momentum of a system of objects is conserved when there is no net force on the system.
- Science: HS-PS2-4 Use mathematical representations of Newton's Law of Gravitation and Coulomb's Law to describe and predict the gravitational and electrostatic forces between objects.

Highlighted Career Ready Practices and 21st Century Themes and Skill

- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas.

<table>
<tr><td colspan="5">• 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions</td></tr>
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• 2.1.12.EH.3: Describe strategies to appropriately respond to stressors in a variety of situations (e.g., academics, relationships, shootings, death, car accidents, illness).</td></tr>
<tr><td colspan="2">Pre-Assessment
• Not Applicable</td><td colspan="3">Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)
• Not Applicable</td></tr>
<tr><td>Student Learning Objectives: We are learning to/that...</td><td>Student Strategies (Mathematical Practices)</td><td>Formative Assessment</td><td>Activities and Resources</td><td>Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)</td></tr>
<tr><td>Find the amount of work done by a constant or variable force.</td><td>SMP 1 Make sense of problems and persevere in solving them.</td><td>How much work is done in pumping water to the top of a conical tank with radius 4 feet and height 6 feet?</td><td>Teacher-led discussion on definition of work and laws of physics applicable for this objective; group tasks to model and solve complex work problems.</td><td>Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.</td></tr>
<tr><td>Find connections between different topics and branches of Calculus to solve or explore advanced problems.</td><td>SMP 1 Make sense of problems and persevere in solving them.</td><td>Show that the function represented by the power series is a solution of the differential equation:
$y = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$
$y'' + y = 0$</td><td>Long-term project:
ie: Calculus of the Normal Distribution - connecting numerical integration, infinite series, and introducing Multivariable Calculus</td><td>Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.</td></tr>
</table>

Research and solve "classic" Calculus problems	SMP 1 Make sense of problems and persevere in solving them.	Model Zeno's Paradox and the Pursuit Problem and find solution(s) using ideas of Calculus.	Zeno's Paradox - Achilles and the Tortoise Pursuit Problem - The Calculus of Friendship (Strogatz)	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Use computer programming to visualize Calculus concepts	SMP 1 Make sense of problems and persevere in solving them.	Solve a general Euler's Method problem symbolically (mapping of program).	Project: Write a program or Desmos file to visualize the concept of Euler's Method.	Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.
Benchmark Assessment • Cumulative Benchmark		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		
Summative Assessment(s) • Quiz on Work by integration **Performance Assessment**: • Cumulative Project		**Modifications/Accommodations (ELL, Special Education, Gifted, At-Risk of Failure, 504)** • Extended time, use of calculator, challenge work and specific other accommodations/modifications per a student's IEP or 504 plan.		

Bibliography
AP Calculus BC

Supplemental Materials/Resources:

Larson, R., Edwards, B. H. (2018). *Calculus AP Edition*. Boston: Cengage Learning.

Digital Textbook and Materials:

www.webassign.com